On the Effect of I/Q Imbalance on Energy Detection and a Novel Four-Level Hypothesis Spectrum Sensing

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***Abstract*— Direct-conversion transceivers are in demand, due to low implementation cost of analog front-ends. However, these transmitters or receivers introduce imperfections such as in-phase and quadrature-phase (I/Q) imbalances. In this paper, we first investigate the effect of I/Q imbalance on the performance of primary system, and show that these impairments can severely degrade the performance of cognitive radio system that are based on orthogonal frequency division multiplexing (OFDM) multiple access scheme. Next, we design a new four-level hypothesis blind detector for spectrum sensing in such cognitive radio system, and show that the proposed detector is less vulnerable to I/Q imbalance than conventional two-level detectors.**

***Index Terms*— Cognitive radio, I/Q imbalance, blind detection, fading channels, OFDMA**

I. Introduction

Cognitive radio is a promising approach to alleviate the spectrum scarcity faced by today wireless communication systems. The spectrum utilization has been noticed to be less than 10% in practice [1], and thus leads to the idea of reusing spectrum by a secondary network [2]. In a cognitive radio network, secondary or unlicensed users benefit by opportunistically accessing the spectrum, while primary or licensed users are protected from the interference.

The recent development in integrated circuit (IC) technology and incorporation of complete phase-locked loop devices in low-cost IC packages have made direct-conversion transceivers widely accepted. Nevertheless, in contrary to their desirable characteristics, the implementation of direct-conversion transceivers causes various impairments associated with the analog components. The in-phase and quadrature-phase (I/Q) imbalance has been known as a major source of analog impairments in high-speed wireless communication systems [3].

The effect of I/Q imbalance on traditional communication systems have been comprehensively investigated in previous works [4]–[10], and various compensation algorithms have been proposed [11]–[14]. However, many of the proposed algorithms cannot be applied to cognitive radio systems. For instance, a compensation method for I/Q distortions based on a novel pilot pattern is proposed in [14]. Since there is no cooperation between the primary transmitter and the secondary receiver in a cognitive radio network, this method and other algorithms evolved out of channel estimation techniques are not applicable in a cognitive radio system. Although the negative effects of I/Q imbalance can be severe, there has not been sufficient studies on the impact of I/Q imbalance in cognitive radio system, and the way to mitigate it. In [15], [16], Neyman-Pearson detector [17] have been exploited to perform spectrum sensing in the presence of transceiver I/Q imbalance. Our work differs as we propose a multi-level detector and study the problem through minimum average cost detection [18] criterion.

In this paper, we consider the effect of interference imposed by I/Q imbalance on the performance of the primary system with blind spectrum sensing at the secondary user. By blind spectrum sensing, we mean that the secondary user does not have access to the instantaneous channel state information of the primary link, which is the same assumption as in [19]. In order to tackle the effects of I/Q imbalance, we propose a four-level hypothesis blind detector *for the first time*, and compare the performance of the proposed detector to the conventional two-level detector under the scenario of I/Q imbalance impaired transceivers.

The subsequent sections of this paper are organized as follows. Section II describes the system model under the effect of I/Q imbalance and explains the idea of utilizing periodogram detector. In Section III, we analyze the effects of secondary transmitter I/Q imbalance on outage probability of the primary system. In Section IV, we explain how I/Q imbalance can affect the spectrum sensing by the secondary user. A four-level hypothesis blind detector is proposed to address the problems of I/Q imbalance, and the detection problem is formulated. Section V compares the performance of the new detector with conventional two-level detectors. Conclusion is given in Section VI.

II. System Model

We consider a cognitive radio network where the primary system utilizes OFDMA technique for its uplink transmission. We assume an overlay cognitive radio system in which the secondary user should not interfere with primary user channels and access only vacant channel. The primary system consists

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of U primary users and dedicates total number of K subcarriers to them. Each primary user transmits a set of designated data symbols $s_{u,k}$ for $k \in \{-K/2, -K/2+1, ..., K/2\}$ and $u \in \{1, 2, ..., U\}$, where $s_{u,k}$ is taken from an *M*-PSK symbol constellation and only one user transmit on a single subcarrier, i.e., orthogonal multiuser transmission. Without loss of generality, we discard the user index *u*, since we assume the same modulation scheme for all primary users, i.e., *M*-PSK symbol constellation. The average power of each subcarrier is assumed to be P_k, $k \in \{-K/2, -K/2+1, ..., K/2\}$. We assume that no data is transmitted on the direct current (DC) subcarrier and *K* is an even number. The cyclic prefix is assumed to be longer than the impulse response of the channel and hence, there is no inter-carrier interference. In addition, we assume that the secondary user knows *a priori* information about potential primary systems including frame structure, subcarrier structure like the Discrete Fourier Transform (DFT) size, the transmission parameters (fundamental symbol rate, cyclic prefix length), and the level of interference tolerance of primary systems [20]. After removing the cyclic prefix and DFT operation, the channel in the subcarrier *k* can be represented as a complex channel h_k, which is modeled as Rayleigh fading with independent Gaussian distributed random variable of variance $\sigma^2_{h_k}$. It has been shown that I/Q imbalance in OFDM-based systems results in a mutual interference between subcarriers that are located symmetrically to the DC carrier [21], [22]. In this paper, we focus on transmitter's I/Q imbalance, since the direct-conversion architecture is commonly employed in transmitters [23]. We assume that both primary and secondary direct-conversion transmitters are wideband, in which the inter user interference imposed by I/Q imbalance is probable.

In practice, there is neither ideal amplitude matching nor exact $\pi/2$ phase difference among I and Q signals. Thus, the actual transmitted signal x_k, which is different from the ideal transmitted signal, can be modeled as [24]:

$$x_k = \alpha_T \sqrt{P_k} s_k + \beta_T \sqrt{P_{-k}} s^{\dagger}_{-k}, \tag{1}$$

where $(.)^{\dagger}$ denotes the complex conjugate. Moreover, the signals sent by primary users are assumed to be normalized, i.e., $\mathrm{E}\{|s_k|\} = \mathrm{E}\{|s_{-k}|\} = 1$. From (1), at the primary transmitter, there is an interference induced by I/Q imbalance, which can be a significant performance limiting factor, especially in the mobile uplinks. The degree of I/Q imbalance can be evaluated in terms of an image rejection ratio (IRR), which is defined by $A_T = \frac{|\beta_T|^2}{|\alpha_T|^2}$ and $A_R = \frac{|\beta_R|^2}{|\alpha_R|^2}$ for transmitter and receiver, respectively [25]. Parameters α_T and β_T in (1) are two complex scalars given by

$$\alpha_T = \cos(\theta_T) + j\epsilon_T \sin(\theta_T), \ \beta_T = \epsilon_T \cos(\theta_T) - j \sin(\theta_T), \tag{2}$$

and ϵ_T and θ_T represent the amplitude and phase mismatch of transmitter I/Q signals, respectively [26]. In addition, α_R and β_R are defined based on (2), but with respect to the receiver mismatch parameters. From now, we use subscripts T, p and T, s to differentiate between the transmit I/Q imbalance parameters of the primary user and that of the secondary user, respectively. From (1), the received signal at secondary

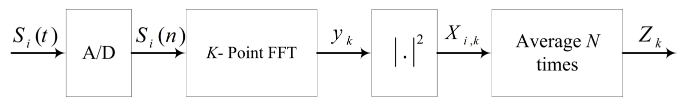

Fig. 1. Block diagram of a priodogram detector for OFDM-based spectrum sensing.

receiver can be written as

$$y_k = \alpha_{T,p} h_k \sqrt{P_k} s_k + \beta_{T,p} h_k \sqrt{P_{-k}} s^{\dagger}_{-k} + \omega_k, \tag{3}$$

where ω_k is the noise with independent and identically distributed (i.i.d.) Gaussian distribution with zero mean and variance σ^2_n, i.e., $\omega_k \sim N(0, \sigma^2_n)$.

A typical energy detector consists of a low pass filter to remove the out-of-band noise and adjacent interference, an analog to digital converter, as well as a square law device to compute the energy. However, this implementation computes the energy of signal within the entire frequency band that is not a preferable approach to perform the spectrum sensing [27]. Hence, a periodogram solution is proposed in [27], which is depicted in Fig.1. In this figure, indices i and k represent the time domain and frequency domain, respectively. From (3), the output of periodogram detector is given by

$$Z_k = \frac{1}{N} \sum_{i=1}^{N} |y_{k,i}|^2 = \frac{1}{N} \sum_{i=1}^{N} X_{k,i} = \frac{1}{N} \sum_{i=1}^{N} (y^2_{R,i} + y^2_{I,i}) \tag{4}$$

where y_R and y_I denote real and imaginary parts of y_k, respectively. In addition, variables N and $X_{k,i}$ represent the number of time-domain data packets and the square-law detector's output for the *i*-th data packet of *k*-th subcarrier, respectively.

III. I/Q Imbalance Effect on Primary System Performance Metric

In this section, we analyze the effect of interference imposed by secondary user on the performance of primary system. This interference occurs due to the I/Q imbalance of the secondary user transmitter. Fig. 2 demonstrates a cognitive radio network with an OFDMA uplink transmission of primary users. Meanwhile, the secondary user senses subcarrier *k* in order to decide if it is vacant. In this example, neither primary user 1 nor primary user 2 transmits data on subcarrier *k*, while the primary user 2 transmits over subcarrier -*k*. The two-level energy detector's decision for subcarrier k is based only on the signal's averaged power in k-th subcarrier. In fact, although the secondary user may be aware of the transmit I/Q imbalance, it just incorporates the information derived from the k-th output of the FFT block for deciding on subcarrier k, regardless of the state of subcarrier $-k$. Hence, if the secondary user uses two-level detector for spectrum sensing, it would transmit over subcarrier *k*, then as a result, primary user 2 would be impaired by the I/Q imbalance from secondary user. As shown in Fig. 2, the secondary transmission on an idle subcarrier, e.g. subcarrier k, introduces an interference on subcarrier $-k$. The primary receiver treats the secondary interference as noise. Hence, the received signal at the primary receiver is

$$y_{p,-k} = \sqrt{P_{-k}} s_{-k} g_{-k} + \beta_{T,s} \sqrt{P_0} s^{\dagger}_{s,k} h_{-k} + w_p \tag{5}$$

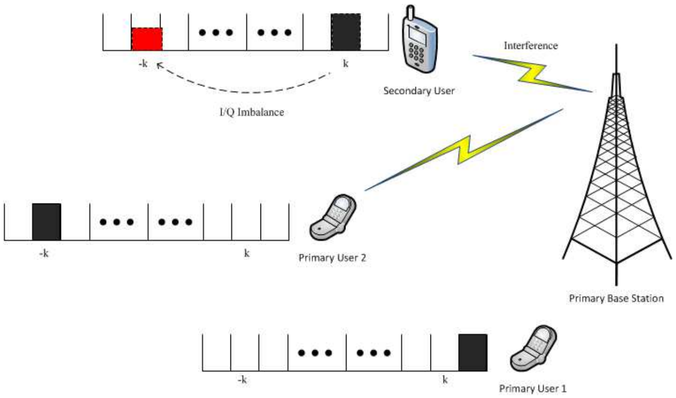

Fig. 2. An example of a secondary user, interfering with OFDMA primary users uplink transmission due to I/Q imbalance caused by cognitive radio system transceivers.

where g_{-k} denotes the channel coefficient of the primary link on subcarrier $-k$, P_0 is the transmitted power of the secondary user, and $s_{s,k}$ is secondary signal on subcarrier k. Moreover, w_p is the additive Gaussian noise of the primary receiver with zero mean and variance of N_p, i.e., $w_p \sim N(0, N_p)$ and is independent of channel coefficients. Assuming a primary uplink transmission with a fixed transmission rate of R_p, the outage probability is defined as $\rho_0 \triangleq \{r_p < R_p\}$ [28]. From (5), and by considering the normalized bandwidth, the data rate r_p is given by

$$r_p = \log_2(1+\gamma) = \log_2\left(1 + \frac{P_{-k}|g_{-k}|^2}{N_p + |\beta_{T,s}|^2 P_0 |h_{-k}|^2}\right), \tag{6}$$

where γ denotes the signal to interference plus noise ratio (SINR). The outage probability can then be expressed as $\rho_0 = \Pr\{\gamma < \gamma_{th}\}$, where $\gamma_{th} = 2^{R_p} - 1$. Hence,

$$\begin{aligned}\rho_0 &= \Pr\left\{\frac{X_1}{1+X_2} < \gamma_{th}\right\} \\ &= \int_{-\infty}^{\infty} \Pr\{X_1 < \gamma_{th}(1+X_2)|X_2\} f_{X_2}(x_2) dx_2, \end{aligned} \tag{7}$$

where X_1 and X_2 are two exponential random variables with mean $\sigma^2_{X_1} = \frac{P_{-k}}{N_p}\sigma^2_{g_{-k}}$ and $\sigma^2_{X_2} = \frac{|\beta_{T,s}|^2 P_0}{N_p}\sigma^2_{h_{-k}}$, respectively. Therefore, the outage probability at the primary receiver can be calculated as

$$\begin{aligned}\rho_0 &= \int_0^{\infty}\left(1 - e^{-\frac{\gamma_{th}(1+x_2)}{\sigma^2_{X_1}}}\right)\left(\frac{1}{\sigma^2_{X_2}} e^{-\frac{x_2}{\sigma^2_{X_2}}}\right) dx_2 \\ &= 1 - \left(\frac{\sigma^2_{X_1}}{\sigma^2_{X_1} + \sigma^2_{X_2}}\right) e^{-\frac{N_p}{\sigma^2_{X_1}}\gamma_{th}}. \end{aligned} \tag{8}$$

IV. Four-Level Hypothesis Test for I/Q Imbalance Impaired Spectrum Sensing

In this section, we introduce a four-level hypothesis test, and apply Bayesian minimum average cost criterion to this model [18]. To understand the motivation of the proposed four-level hypothesis test, again consider the scenario depicted in Fig. 2.

In addition to the problem discussed in previous section, I/Q imbalances imposed by primary users can make a secondary user assume a vacant subcarrier busy. This would rise the probability of false alarm and degrades the system performance. Therefore, we use this notion to propose a four-level hypothesis test, where the detector can adjust the decision thresholds more intelligently and detect vacant subcarriers more precisely. We model the four possible cases that could happen at a secondary receiver as follows:

$$\begin{cases} \mathrm{H}_0\text{: Only Noise} \\ \mathrm{H}_1\text{: I/Q Imbalance+Noise} \\ \mathrm{H}_2\text{: Primary user signal+Noise} \\ \mathrm{H}_3\text{: Primary user signal+I/Q Imbalance+Noise.} \end{cases}$$

The first state occurs when the secondary user receives only noise and there is no data transmission by any of the primary users in both *k*-th and -*k*-th subcarriers. The second state happens when there is transmission in subcarrier -*k* by one of the primary users, while subcarrier *k* is left vacant. Hence, the secondary user senses the interference plus noise, where the interference is due to the I/Q imbalance of secondary receiver or primary transmitter or both of them. If primary user transmits in subcarrier *k*, then secondary user senses only primary signal plus noise as stated by the third hypothesis. The forth state stands for the transmission of data over both *k*-th and -*k*-th subcarriers by primary users. Therefore, at this state the secondary user receives a signal associated with the *k*-th subcarrier and also an interference plus noise.

The proposed detector is able to differentiate the interference imposed by mirrored subcarrier from the main signal, i.e., by detecting H_1. This is important, because secondary user can use this group of subcarriers in case that secondary transmitter does not suffer from considerable I/Q imbalance. Even for the secondary user with high transmit I/Q imbalance, this detector helps user to be aware of the primary transmission on subcarrier $-k$ and avoid interfering with primary system. According to the four-level hypothesis model and the expression for the received signal from (3) and (4), the hypothesis test can be formulated as

$$\begin{cases} \mathrm{H}_0 : Z_k = \frac{1}{N}\sum_{i=1}^{N} |\omega_{k,i}|^2 \\ \mathrm{H}_1 : Z_k = \frac{1}{N}\sum_{i=1}^{N} |(\beta_{T,p} h_{k,i})\sqrt{P_{-k,i}} s^{\dagger}_{-k,i} + \omega_{k,i}|^2 \\ \mathrm{H}_2 : Z_k = \frac{1}{N}\sum_{i=1}^{N} |(\alpha_{T,p} h_{k,i})\sqrt{P_{k,i}} s_{k,i} + \omega_{k,i}|^2 \\ \mathrm{H}_3 : Z_k = \frac{1}{N}\sum_{i=1}^{N} |(\alpha_{T,p} h_{k,i})\sqrt{P_{k,i}} s_{k,i} \\ \qquad\qquad + (\beta_{T,p} h_{k,i})\sqrt{P_{-k,i}} s^{\dagger}_{-k,i} + \omega_{k,i}|^2. \end{cases} \tag{9}$$

From (3), it is obvious that the received signal y_k is a complex random variable. Hence, $y_k = y_{R,k} + j y_{I,k}$ where

$$\begin{aligned} y_{R,k} &= Re\{\alpha_{T,p}\sqrt{P_k} s_k + \beta_{T,p}\sqrt{P_{-k}} s^{\dagger}_{-k}\} h_{R,k} \\ &\quad - Im\{\alpha_{T,p}\sqrt{P_k} s_k + \beta_{T,p}\sqrt{P_{-k}} s^{\dagger}_{-k}\} h_{I,k} + \omega_{R,k}, \\ y_{I,k} &= Re\{\alpha_{T,p}\sqrt{P_k} s_k + \beta_{T,p}\sqrt{P_{-k}} s^{\dagger}_{-k}\} h_{I,k} \\ &\quad + Im\{\alpha_{T,p}\sqrt{P_k} s_k + \beta_{T,p}\sqrt{P_{-k}} s^{\dagger}_{-k}\} h_{R,k} + \omega_{I,k}, \end{aligned} \tag{10}$$

and $\omega_{R,k}$ and $\omega_{I,k}$ represent real and imaginary parts of ω_k, respectively. It can be checked from (10) that real and imaginary parts of y_k are Gaussian random variables with zero mean and identical variances. In addition, $y_{R,k}$ and $y_{I,k}$ are uncorrelated [29]. Hence, they are independent and y_k can be represented as a complex Gaussian random variable. Thus, the probability density function of their square is given by [30]

$$f_{y_R^2}(x) = f_{y_I^2}(x) = \begin{cases} \frac{1}{\sqrt{2\pi x \sigma^2}} e^{-\frac{x}{2\sigma^2}} & \text{if } x \geq 0, \\ 0 & \text{if } x < 0, \end{cases} \quad (11)$$

where $f_X(.)$ and σ^2 denote the probability density function of variable X and the variance of either real or imaginary part of y_k, respectively. In addition, considering the fact that y_R^2 and y_I^2 are independent, and from (11), the probability density function of X_k is

$$f_{X_k}(x) = \int_{-\infty}^{\infty} f_{y_R^2}(x-s)\, f_{y_I^2}(s)\, ds = \int_0^x \frac{1}{2\pi\sigma^2} \frac{e^{-\frac{x}{2\sigma^2}}}{\sqrt{s(x-s)}} ds$$
$$= \frac{1}{2\pi\sigma^2} e^{-\frac{x}{2\sigma^2}} \int_0^1 \frac{2}{\sqrt{1-u^2}} du = \frac{1}{2\sigma^2} e^{-\frac{x}{2\sigma^2}}. \quad (12)$$

From (12), the conditional probability density function of X_k can be represented as exponential distribution as follows:

$$f_X(x|H_i) = \begin{cases} \frac{1}{2\sigma_i^2} e^{-\frac{x}{2\sigma_i^2}} & \text{if } x \geq 0, \\ 0 & \text{if } x < 0, \end{cases} \quad (13)$$

where $i = 0, 1, 2, 3$ and

$$\begin{cases} \sigma_0^2 = \frac{1}{2}\sigma_n^2 \\ \sigma_1^2 = \frac{1}{2}(|\beta_{T,p}|^2 P_{-k} |s_{-k}|^2 \sigma_{h_k}^2) + \sigma_0^2 \\ \sigma_2^2 = \frac{1}{2}(|\alpha_{T,p}|^2 P_k |s_k|^2) \sigma_{h_k}^2 + \sigma_0^2 \\ \sigma_3^2 = \frac{1}{2}(|\alpha_{T,p}|^2 P_k |s_k|^2 \\ \qquad + 2Re(\alpha_{T,p}\beta_{T,p}\sqrt{P_k}\sqrt{P_{-k}} s_k s_{-k}^\dagger))\sigma_{h_k}^2 + \sigma_1^2 . \end{cases} \quad (14)$$

The periodgram detector, introduced in Section II, computes the average of FFT output for N time domain windowed data packets for each frequency slot. The overall output of the detector for each frequency slot is a Gamma distributed random variable, since it is a sum of N exponential random variables with identical means, i.e., $Z_k = \frac{1}{N}\sum_{i=1}^{N} |y_{i,k}|^2$. Therefore, we have

$$f_{Z_k}(z|H_i) = \begin{cases} \frac{1}{\Gamma(N)} (\frac{1}{N\sigma_i^2})^N z^{(N-1)} e^{-\frac{z}{N\sigma_i^2}} & \text{if } z \geq 0, \\ 0 & \text{if } z < 0, \end{cases} \quad (15)$$

where $\Gamma(.)$ is the gamma function [31, Sec. (8.31)].

Now, we apply the minimum average cost criterion to this model. Without losing any generality, a uniform cost test with equal prior probabilities is assumed for all hypotheses. Thus, H_i is selected if [18]

$$\frac{f_{Z_k}(z|\mathrm{H}_i)}{f_{Z_k}(z|\mathrm{H}_j)} > 1 \text{ for } i, j = 0, 1, 2, 3 \text{ and } i \neq j\, . \quad (16)$$

It can be shown that by applying the conditions in (16), four decision regions are obtained as follows:

$$\begin{cases} \Lambda_0 : \{0 < z < S_{01}, 0 < z < S_{02}, 0 < z < S_{03}\} \\ \Lambda_1 : \{S_{01} < z < S_{12}, S_{01} < z < S_{13}\} \\ \Lambda_2 : \{S_{12} < z < S_{23}, S_{02} < z < S_{23}\} \\ \Lambda_3 : \{S_{03} < z, S_{13} < z, S_{23} < z\} \quad , \end{cases} \quad (17)$$

where Λ_i is the corresponding region of each hypothesis and

$$S_{ij} = S_{ji} = \frac{N^2 \ln \frac{\sigma_i^2}{\sigma_j^2}}{\frac{1}{\sigma_j^2} - \frac{1}{\sigma_i^2}} \text{ for } i, j = 0, 1, 2, 3 \text{ and } i \neq j. \quad (18)$$

In Appendix A, it is shown that $S_{01} < S_{02} < S_{03}$, $S_{02} < S_{12} < S_{13}$, and $S_{03} < S_{13} < S_{23}$. Thus, the expressions in (17) can be simplified as:

$$\Lambda_0 : \{0 < z < S_{01}\}, \quad \Lambda_1 : \{S_{01} < z < S_{12}\},$$
$$\Lambda_2 : \{S_{12} < z < S_{23}\}, \ \Lambda_3 : \{0 < S_{23} < z\}. \quad (19)$$

In order to analyze the effect of I/Q imbalance, we calculate the probability of false alarm, i.e., when there is no signal transmission in subcarrier k , but secondary user assumes that the channel is busy. Equivalently,

$$P_{fa} = P(\mathrm{H}_2|\mathrm{H}_1) + P(\mathrm{H}_2|\mathrm{H}_0) + P(\mathrm{H}_3|\mathrm{H}_1) + P(\mathrm{H}_3|\mathrm{H}_0)$$
$$= \int_{\Lambda_2} [P(z|\mathrm{H}_1) + P(z|\mathrm{H}_0)]\, dz + \int_{\Lambda_3} [P(z|\mathrm{H}_1) + P(z|\mathrm{H}_0)]\, dz. \quad (20)$$

From (15), the integrals can be evaluated using the following:

$$P(\mathrm{H}_3|\mathrm{H}_i) = \int_{\Lambda_3} P(z|\mathrm{H}_i) = \int_{S_{23}}^{\infty} \frac{1}{\Gamma(N)} (\frac{1}{N\sigma_i^2})^N z^{(N-1)} e^{-\frac{z}{N\sigma_i^2}} dz$$
$$= \frac{1}{\sigma_i^2 \Gamma(N+1)} \int_{S_{23}}^{\infty} (\frac{z}{N\sigma_i^2})^{N-1} e^{-(\frac{z}{N\sigma_i^2})} dz$$
$$= \frac{1}{\Gamma(N)} \int_{\frac{S_{23}}{N\sigma_i^2}}^{\infty} t^{N-1} e^{-t} dt = \frac{\Gamma(N, \frac{S_{23}}{N\sigma_i^2})}{\Gamma(N)}, \quad (21)$$

where $\Gamma(.,.)$ is the incomplete gamma function [31]. Therefore, from (20) and (21) the probability of false alarm can be expressed as

$$P_{fa} = \frac{\Gamma(N, \frac{S_{12}}{N\sigma_1^2}) - \Gamma(N, \frac{S_{23}}{N\sigma_1^2})}{\Gamma(N)} + \frac{\Gamma(N, \frac{S_{12}}{N\sigma_0^2}) - \Gamma(N, \frac{S_{23}}{N\sigma_0^2})}{\Gamma(N)}$$
$$+ \frac{\Gamma(N, \frac{S_{23}}{N\sigma_1^2})}{\Gamma(N)} + \frac{\Gamma(N, \frac{S_{23}}{N\sigma_0^2})}{\Gamma(N)} = \frac{\Gamma(N, \frac{S_{12}}{N\sigma_1^2})}{\Gamma(N)} + \frac{\Gamma(N, \frac{S_{12}}{N\sigma_0^2})}{\Gamma(N)}. \quad (22)$$

Similarly, the probability of detection is given by

$$P_D = P(\mathrm{H}_2|\mathrm{H}_2) + P(\mathrm{H}_3|\mathrm{H}_3)$$
$$= \frac{\Gamma(N, \frac{S_{12}}{N\sigma_2^2})}{\Gamma(N)} - \frac{\Gamma(N, \frac{S_{23}}{N\sigma_2^2})}{\Gamma(N)} + \frac{\Gamma(N, \frac{S_{23}}{N\sigma_3^2})}{\Gamma(N)} \ . \quad (23)$$

It must be noted that different side information is needed in order to calculate the detection thresholds. For instance, the statistics of the interference link between primary users and secondary users, i.e. $\sigma_{h_k}^2$, could be inferred by listening to the downlink transmission of the primary system [28]. In addition, the determination of primary transmitter IRR values or transmit power requires either explicit signaling from the primary system to the secondary users, or the primary transmit power could be estimated if secondary user roughly knows about its distance from the nearby primary user. This work could also be extended to MIMO systems in which localization and other array-antenna techniques are incorporated.

V. Simulation Results

In this section, we provide simulations to evaluate the analytical results provided in previous sections. We assume a cognitive radio network with an OFDMA primary system with four users, i.e., $U = 4$. Moreover, the total number of subcarriers is $K = 512$, i.e., 128 subcarrier per user with 16-PSK constellation. We also assume a secondary user that uses blind detection for spectrum sensing, based on the four-level hypothesis test proposed in Section IV. The magnitude

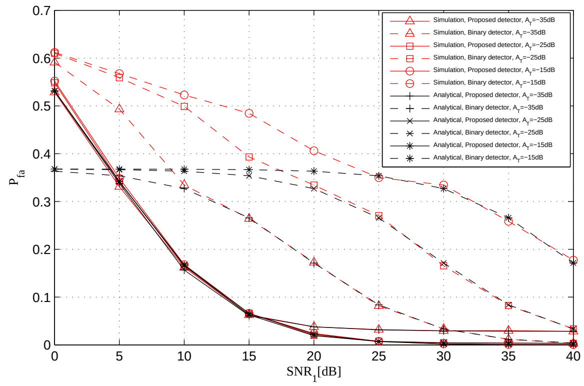

Fig. 3. The comparison between the probability of false alarm of the proposed four-level detector and the conventional two-level detector for different values of IRRs. The dashed and solid curves represent the probability of false alarm for conventional detector and proposed detector, respectively. The results are obtained for different values of IRRs and for the constant transmitted power of subcarrier $-k$, i.e., $\text{SNR}_2 = -10dB$.

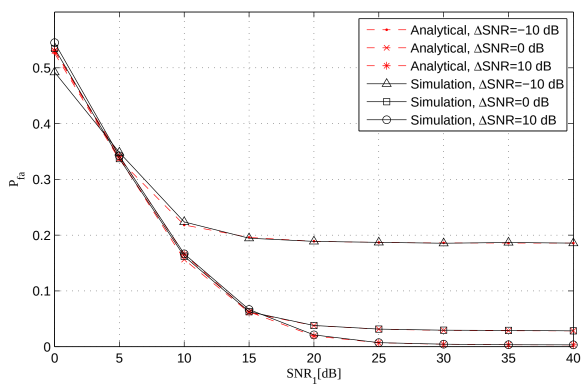

Fig. 4. The probability of false alarm for different ΔSNR values, when IRR= -15dB. The dashed and solid curves represent the analytical results and the simulations, respectively.

of the frequency domain channel coefficients h_k and h_{-k} are assumed independent Rayleigh distributed random variables with variance $\sigma^2_{h_k} = \sigma^2_{h_{-k}} = 1$. In order to examine the statistical aspects of energy detector, simulations are done over 20 million samples and for one sample package, i.e., $N = 1$. Here, we define the transmitted signal to noise ratio for subcarrier k and $-k$ as $\text{SNR}_1 = \frac{P_k|s_k|^2}{\sigma^2_n}$ and $\text{SNR}_2 = \frac{P_{-k}|s_{-k}|^2}{\sigma^2_n}$, respectively. We consider orthogonal signaling, hence interference is only due to the I/Q imbalance. Secondary receiver treats primary interference as the Gaussian noise, since it is not completely aware of the primary signal's modulation parameters. The SNR at each subcarrier can be determined by priodogram detector described in Section II.

The comparison between the proposed four-level detector and the conventional two-level detector is shown in Fig.3. Based on the results, the proposed detector is less vulnerability to the I/Q imbalance effect than the two-level detector. Since the two-level detector sets its detection threshold based only on the estimated noise variance, the interference imposed by other subcarriers can fool the detector. On the other hand, the proposed detector uses the joint information of both subcarriers k and $-k$, by adding hypotheses H_1 and H_2 into its detection procedure. This helps the detector to sense the spectrum based on the variance of both noise and interference which leads to lower probability of false alarm.

Fig.4 demonstrates both analytical and simulation results of I/Q imbalance effect on probability of false alarm for the proposed detector by considering various $\Delta\text{SNR} = 10\log\frac{\text{SNR}_1}{\text{SNR}_2}$, which denotes the signal to noise ratio difference between the subcarriers k and $-k$. Fig.4 shows that even for $\Delta\text{SNR} = -10$dB, the probability of false alarm is significantly increased.

The comparison of joint transmitter-receiver I/Q imbalance effect on the probability of detection with the case when I/Q imbalance exists only in transmitters is shown in Fig. 5. The joint transmitter-receiver I/Q imbalance effect can be investigated based on the procedure introduced in this paper. Indeed, it can be proven that the variance of received signal may differ from that of presented in Section IV, nevertheless the detector output distributions are identical in both scenarios. Hence, the probability of false alarm and the probability of detection could be found accordingly. As expected from analytical results, effect of I/Q imbalance on the receiver's probability of detection is serious when direct-conversion transceivers are used. However, direct-conversion receivers offer significant power saving, hence they might be as preferable as direct-conversion transmitters to be utilized in wireless communication devices [23].

Eventually, Fig. 6 depicts the impact of I/Q imbalance of the secondary transmitter on the outage probability of primary receiver. It clearly demonstrates that the I/Q imbalance in secondary transmitter can dramatically affect the performance of the primary system.

VI. Conclusion

In this paper, the effects of I/Q imbalance on both primary and secondary systems have been investigated. It was explained how I/Q imbalance can make the secondary user interfere to an OFDMA primary system if the secondary user employs binary detector. Therefore, we design a new detector based on four-level hypothesis for blind spectrum sensing in an overlay cognitive radio system. The detection algorithm jointly considers the information of DFT blocks output for a specific subcarrier and the interference imposed by its symmetrical counterpart. Simulation results show that the proposed detector decreases the probability of false alarm, while avoids interfering to primary system.

Appendix A

From (14) and assuming $\lambda_i = \frac{1}{\sigma_i^2}$ as well as $\sigma_2^2 > \sigma_1^2$, it is obvious that $0 < \lambda_3 < \lambda_2 < \lambda_1 < \lambda_0$. Moreover, from (18), we have

$$\begin{aligned} S_{ij} = S_{ji} &= N^2\frac{\ln(\lambda_i) - \ln(\lambda_j)}{\lambda_i - \lambda_j} \\ &\triangleq N^2 \frac{\Delta F(x)}{\Delta x}\Big|_{\lambda_i,\lambda_j} \quad \text{for } i,j = 0,1,2,3 \text{ and } i \neq j, \quad (24) \end{aligned}$$

where $F(x) = \ln(x)$. Therefore, (24) suggests that S_{ij} is of the form of $F(x)$ differentiation, i.e., $\frac{1}{x}$, which is a decreasing function of x. Hence, $S_{01} < S_{02} < S_{03}$, $S_{02} < S_{12} < S_{13}$, and $S_{03} < S_{13} < S_{23}$.

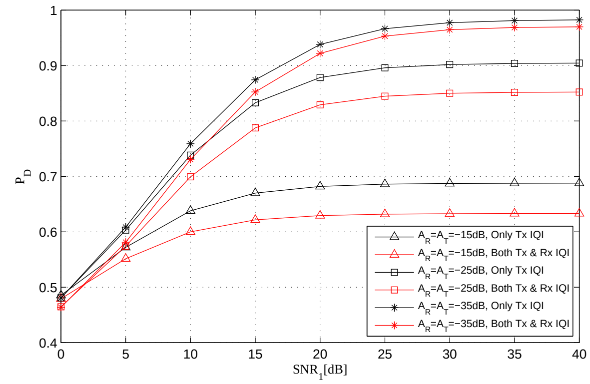

Fig. 5. The comparison of joint transmitter-receiver I/Q imbalance effect on the probability of detection with the case when I/Q imbalance exists only in transmitters for different values of IRRs. The IRR values are assumed to be identical in transmitters and receivers, i.e., $A_T = A_R$.

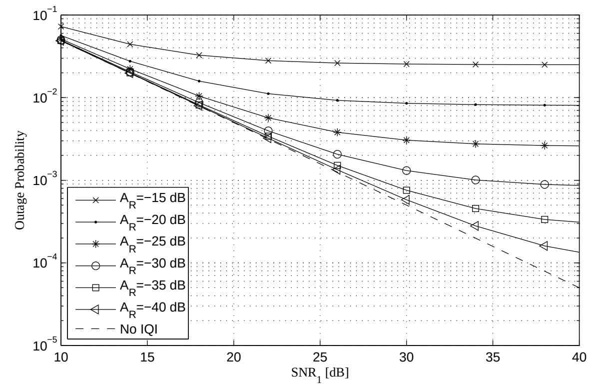

Fig. 6. The effect of I/Q imbalance on outage probability of primary system for different values of IRRs

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